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Cop and robber games when the robber can hide and ride

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In the classical cop and robber game, two players, the cop \mathcal{C} and the robber \mathcal{R} , move alternatively along edges of a finite graph $G = (V, E)$. The cop captures the robber if both players are on the same vertex at the same moment of time. A graph G is called *cop win* if the cop always captures the robber after a finite number of steps. Nowakowski, Winkler (1983) and Quilliot (1983) characterized the cop-win graphs as dismantlable graphs. In this talk, we will characterize in a similar way the class $CWFR(s, s')$ of cop-win graphs in the game in which the cop and the robber move at different speeds s' and s , $s' \leq s$. We also establish some connections between cop-win graphs for this game with $s' < s$ and Gromov's hyperbolicity. In the particular case $s' = 1$ and $s = 2$, we prove that the class of cop-win graphs is exactly the well-known class of dually chordal graphs. We show that all classes $CWFR(s, 1)$, $s \geq 3$, coincide and we provide a structural characterization of these graphs. We also investigate several dismantling schemes necessary or sufficient for the cop-win graphs (which we call *k-winnable* and denote by $CWW(k)$) in the game in which the robber is visible only every k moves for a fixed integer $k > 1$. We characterize the graphs which are *k-winnable* for any value of k .

We present now our main results (for the full version, see [1]). A graph $G = (V, E)$ is (s, s') -*dismantlable* if its vertices can be ordered v_1, \dots, v_n so that for each $v_i, 1 \leq i < n$, there exists v_j with $j > i$, such that $N_s(v_i, G \setminus \{v_j\}) \cap \{v_i, v_{i+1}, \dots, v_n\} \subseteq N_{s'}(v_j)$. A graph G is *dually chordal* if its clique hypergraph is a hypertree. Dually chordal graphs are exactly the graphs $G = (V, E)$ admitting a *maximum neighborhood ordering*.

Theorem 1. *For any $s, s' \in \mathbb{N} \cup \{\infty\}$, $s' \leq s$, a graph $G = (V, E)$ belongs to $CWFR(s, s')$ iff G is (s, s') -dismantlable.*

Theorem 2. *For a graph $G = (V, E)$, the following conditions are equivalent: (i) $G \in CWFR(2)$; (ii) G is $(2, 1)$ -dismantlable; (iii) G is dually chordal.*

A graph G is a *big brother graph* if its blocks can be ordered B_1, \dots, B_r such that B_1 is dominated and, for any $i > 1$, the block B_i is a leaf in the block-decomposition of $\cup_{j \leq i} B_j$ and is dominated by the articulation point connecting B_i to $\cup_{j < i} B_j$. A graph G is a *big two-brother graph* if G can be represented as a union of its subgraphs G_1, \dots, G_r labeled in such a way that G_1 has a dominating vertex, and for any $i > 1$, either the subgraph G_i intersects $\cup_{j < i} G_j$ in two adjacent vertices x_i, y_i belonging to a common subgraph $G_j, j < i$, so that y_i dominates G_i , or G_i has a dominating vertex y_i and intersects $\cup_{j < i} G_j$ in a single vertex x_i (that may coincide with y_i).

Theorem 3. *For a graph $G = (V, E)$ the following conditions are equivalent: (i) $G \in CWFR(3)$; (i') G is $(3, 1)$ -dismantlable; (ii) $G \in CWFR(\infty)$; (ii') G is $(\infty, 1)$ -dismantlable; (iii) G is a big brother graph. In particular, the classes of graphs $CWFR(s), s \geq 3$, coincide.*

Theorem 4. *A graph $G = (V, E)$ is k -winnable for all $k \geq 1$ if and only if G is a big two-brother graph.*

A graph $G = (V, E)$ is k -*bidismantlable* if its vertices can be ordered v_1, \dots, v_n in such a way that for each vertex $v_i, 1 \leq i < m$, there exist two adjacent or coinciding vertices x, y with $y = v_j, x = v_\ell$ and $j, \ell > i$ such that $N_k(v_i, G \setminus \{x, y\}) \cap X_i \subseteq N_1(y)$, where $X_i := \{v_i, v_{i+1}, \dots, v_m\}$.

Theorem 5. *Any graph $G = (V, E)$ of $CWW(2)$ is 2-bidismantlable, however there exist 2-bidismantlable graphs G with $G \notin CWW(2)$. For any odd integer $k \geq 3$, if a graph G is k -bidismantlable, then $G \in CWW(k)$.*